OIL AND GAS TRANSPORTATION AND STORAGE

Replacement of buffer gas with nitrogen in gas storage formations (models, methods, numerical experiments)

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The paper gives description of the object of study - reservoir of the underground gas storage facility. A problem of replacement of buffer gas with nitrogen is raised and the problem formulations for its candidate solution are shown. A mathematical model of replacing buffer gas with nitrogen is proposed, which includes filtering model and convection model - diffusion of gases with concentrated sources. For the cases of unmixing gases the algorithm was developed for finding the propagation path of nitrogen. Numerical experiments were carried out.

Gas storage formations are porous media of different permeability. Filtration of two-component gas in them is accompanied by the convection and diffusion processes. The mathematical model of gas filtration in heterogeneous formations is quite complicated. It is related, in particular, to essential heterogeneity of formations. Another factor is that the diffusion and convection processes create flows of different intensity: convective processes in a porous medium are usually much more intensive than the diffusive ones. As a result, the mathematical models of filtration processes in gas storage formations contain transient nonlinear differential equations in partial derivatives with quick-changing and discontinuous ratios. Most of the parameter problems which can be formulated in the framework of such models cannot be solved analytically, so in order to solve them numerical methods must be applied.

The content of any gas storage can be conditionally divided into two parts – buffer and commercial gas. As distinct from the commercial one, the buffer gas as a rule does not go outside of the storage tank. In the filled gas storage the buffer gas is mostly located in the collector – formation parts with low permeability. Essential escalation in prices for the natural gas actualizes the problem of development of economic feasibility of the technology of replacement of a part of the buffer gas with the cheap gas subject to preservation of existing modes of extraction / injection of the commercial gas and environmental safety.

Development of such technology for each specific gas storage envisages performance of numeric and thus on-site experiments. For performance of numeric experiments the mathematical models describing the physical processes which accompany replacement of the natural buffer gas with another gas or liquid are required. In the framework of such models one can evaluate effect of various physical processes on the process of replacement of the buffer gas, make forecast calculations in considerable time intervals and work out the replacement technologies which would ensure controlled effect of replacement of the natural gas on the existing modes of extraction / injection of the gas in storage, as well as work out various replacement variants. All possible replacement variants developed on the basis of modeling and expert values may be classified according to economic and operating criteria, safety degree, feasibility etc.

One of the main problems which cannot be neglected during modeling of replacement of the natural gas with nitrogen is quantitative evaluation of their mixing taking into account effect of changes in directions of the commercial gas flow (extraction and injection modes) and filtration speed, as well as interdiffusion of the commercial and buffer gases. In case of high intensity of mixing, the commercial gas quality decreases, that's why replacement of the buffer gas with the commercial one may become economically unjustified. The process of mixing of the buffer and commercial gases in heterogeneous formations will essentially depend on intensity of injection of the buffer gas, flows between heterogeneous gas storage formations which depend on the storage operating modes.

Construction of the mathematical model in order to describe the physical processes which accompany replacement of a part of the buffer gas with nitrogen and work out various replacement variants, economical advantageous volumes and paces of replacement requires solving of a range of tasks, in particular:

performance of comparative analysis of gasodynamic characteristics of nitrogen and the natural gas;

study of filtration, diffusion properties and solubility of nitrogen in the natural gas under the real conditions of interaction thereof with each other and the water;

study of dynamic parameters of interaction of gas with porous medium of the formation (study of factors of influence on the interaction dynamics);

study of the processes of mixing of the natural gas and nitrogen during their cofiltration;

performance of analysis of the processes of codiffusion of nitrogen, the natural gas and water;

study of the processes of combustion of the natural gas with various nitrogen concentration and effect of the nitrogen concentration in the natural gas on the mixture heating value;

modeling of the process of extraction / injection of the natural gas with concurrent injection of nitrogen;

planning of the UGSF operating modes with partial replacement of the buffer gas with nitrogen and evaluation of its efficiency for specific collector-layers.

Solving of these tasks is possible in the framework of adequate mathematical models which describe replacement of the buffer gas taking into account the volumes and paces of displacement of the buffer gas with the cheap substitute. Besides, in order to solve nonlinear dynamic problems formulated in the framework of such models, efficient numeric methods must be used.

The works [1–7] are devoted to the problem of modeling of the processes of gas filtration in underground storage facilities. But the filtration problems arising during investigation of replacement of the buffer gas with nitrogen in specific gas storage facilities has presently been studied insufficiently.

This work is intended to construct the mathematical model of two-component isothermal filtration of the gas mixture in the formations which are heterogeneous by permeability, porosity and capacity, formulation in the framework of the model of the problems of replacement of the buffer gas with nitrogen, development of numeric methods of solving thereof and quantitative study of the filtration processes.

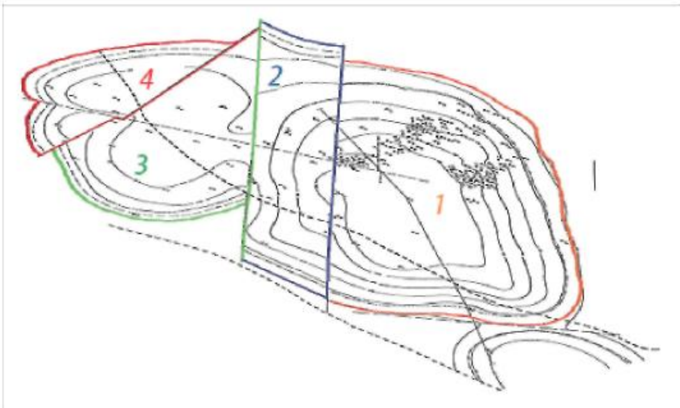

Fig. 1. Structural map with division of the UGSF into activity blocks

Physical study subject – collector-layer

According to the data of the studies performed during operation of the Dashavske underground gas storage facility (UGSF), the gas saturation zone of the facility and the gas volumes accumulated in it can be divided into four blocks (Fig. 1), delimited by the low permeability areas. The first block contains the main operating deposits Г and Е which are directly operated in the gas injection and extraction modes. During cyclic operation, the gas outflows from these deposits to the deposit $Д\text{-}Д_1$ and the next one and vice versa.

The second block is transient from the active zone of the first block to the dead zones. The deposit $Д\text{-}Д_1$ belongs to it.

The third block is located to the south from the tectonic deformation. Arrangement of the deposit Ж+В corresponds to the forth block. Unessential inflow of the gas to it from the second block of the deposit $Д\text{-}Д_1$ is possible.

Presently, interaction between the deposits became stable, which stipulates stable operation of the storage facility in general. The gas deposits Г, Е, Д and Ж+В in the aggregate make up a uniform gas-hydrodynamic system and are operated as one object of the gas facility.

Study subject parameters

The total porous volume of the UGSF makes up 117.4 mln m^3, including the deposits Е+Г+Д – 108.5 mln m^3, deposits Ж+В – 8.9 mln m^3. The total area of the UGSF deposits (Е+Г+Д) makes up 62,790 thsd. m^2, including the first block – 19,350 thsd. m^2 (30.8 %), the second block – 23,600 thsd. m^2 (37.6 %), the third and the forth blocks – 19,840 thsd. m^2 (31.6 %).

Analyzing propagation of the gas among the blocks at the end of the extraction (as of 01.04.2002) and injection seasons (as of 01.10.2002), we make the conclusion that the third and the forth blocks are "replacement" zones with the gas reserves of about 460 mln m^3, which practically did not change during the cycle. In the second block the "dead" area contains about 1,500 – 1,600 mln m^3 of the natural gas. On this basis we can make the conclusion that in the dead areas about 1.9 blnд m^3 of the natural gas which likely can be replaced with nitrogen was accumulated.

Mathematical model of filtration-diffusion processes

Nitrogen filtration in the porous medium filled with the natural gas is stipulated by the gas codiffusion and coconvection. As a result, on the edge of the media the gas mixing area appears with the gases which are close by viscosity. Convective diffusion depends on the structure of porous channels. The value of the convective diffusion is affected by pore dimensional variation. Molecular diffusion is determinative in case of low paces of the process of displacement of the natural gas by nitrogen. In case of the gas flow paces of about 3-4 m/sec the ratios of the molecular diffusion and convective diffusions are of the same magnitude. In case of high paces of the gas filtration, the convective diffusion ratio is by 1-2 orders more than the molecular diffusion ratio. As the gas turbulence degree increases the molecular diffusion value decreases practically to nothing.

Formulating the mathematical model of replacement of the buffer natural gas with nitrogen we will proceed from the fact that according to gasodynamic characteristics the natural gas and nitrogen differ insignificantly. So, it should be expected that their permeability in porous media also differs insignificantly. It gives us the ground to combine the problems of filtration of both gases into one filtration problem. Such assumptions must not essentially affect the modeling results, since uncertainties as to the main physical and geometrical parameters of the formation are more essential.

In terms of such assumption, in case is the natural gas and the substitute are not mixed, the problem resolves itself to finding of the gas propagation limit at every moment of time of the replacement process. In other case one must determine the nitrogen (or the natural gas) concentration) in the gas mixture generated during the replacement process as the function of spatial coordinates and time.

We will calculate the gas-substitute volumes in the formation medium using the nitrogen state equation. The nitrogen compressibility factor under the real formation conditions will be close to 0.97.

Mathematical model of gas filtration. The gas storage facility formation is the geological formation which occupies a certain three-dimensional area $\Omega \subset R^3$ and is characterized by a certain width and lateral dimensions. In order to describe the gas flows in the formation, let's introduce the Cartesian coordinate system *{x, y, z}* with the directed vertically (conversely to the gravitation forces) axis *Oz,*. The width *h(x, y)* of the formation is much lesser than its other geometric dimensions. That's why the vertical fluctuations of the values of gasodynamic parameters are insignificant, and, thus, we can neglect their dependence on the width coordinate and study the formation as a two-dimensional area $\Omega \subset R^2$ limited by the line $\mathbf{\Gamma}$. The well bottom areas $\Omega_i \subset R^2$, $i = 1, ..., n,$ whose sizes are small in comparison with the sizes of the area $\Omega \subset R^2$ will not exclude the area Ω.. Then the line $\mathbf{\Gamma}$ delimiting the formation consists of the line Γ_i. delimiting the bottom area Ω_i and the line Γ_z determining the external limit of the formation:

$$\mathbf{\Gamma} = \Gamma_w \cup \Gamma_z, \text{де } \mathbf{\Gamma_w} = \bigcup_{i=1}^{n} \mathbf{\Gamma_i}.$$

The gas propagation pressure *p(x, y, t)* during its filtration process in the porous medium is determined using the equation [3]

$$\frac{\partial}{\partial x}\left[\frac{kh}{\mu z_s}\frac{\partial p^2}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{kh}{\mu z_s}\frac{\partial p^2}{\partial y}\right]=2amh\frac{\partial}{\partial t}\left[\frac{p}{z_s}\right]+2q(t)hp_0, \quad (1)$$

where $k = k(x, y, p)$ and $m = m(x, y)$ – permeability and porosity factors, $q(t)$ – source function; z_s – compressibility factor; μ – dynamic viscosity ratio; p_0 – atmospheric pressure.

Since the value of the formation pressures p_i in the bottoms of operating and supervised wells are known, then the pressures in the areas Ω_i are assumed as given. That's why on the portion $\mathbf{\Gamma}_w$ of the line $\mathbf{\Gamma}$ solution of the equation (1) must be subordinated to the Dirichlet's boundary condition

where $\nu_x = \cos(\nu, x)$, $\nu_y = \cos(\nu, y)$ – components of the external normal vector up to the line Γ_z.

Mathematical model of gas diffusion. The nitrogen concentration propagation $C(x, y, t)$ in the porous medium filled with the natural gas is determined by the equation:

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right)-\frac{\partial(\nu_x C)}{\partial x}-\frac{\partial(\nu_y C)}{\partial y}+U(x,y,C_0,t), \quad (4)$$

where D – molecular diffusion factor, v_x, v_y – components of the gas filtration (convective transfer) pace vector, $U(x, y, t)$ – known function of nitrogen concentrated sources. The molecular diffusion factor can be determined using the known formula [7]

$$D = \frac{10^{-3} T^{1.75}}{P(v_A^{1/3} + v_G^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_G}}, \qquad (5)$$

where T – absolute temperature, P – gas pressure, v_A, $\mathbf{v}_G$ and M_A, M_G – mole volumes and mole weights of nitrogen and the natural gas correspondingly.

Solution of the equation (4) must be subordinated on the line Γ_z to the boundary condition

$$\frac{\partial C}{\partial x} v_x + \frac{\partial C}{\partial y} v_y = 0,\ (x, y) \in \Gamma_z, \qquad (6)$$

which determines permeability for nitrogen of the formation limit Γ_z.

The source function U determines intensity of the gas-substitute through the wells which are used for nitrogen injection into the formation. Since the bottom areas are small as compared to the formation sizes, then we will consider the function U as the totality of concentrated sources, presenting it in the form of

$$U(x_i, y_i, t) = \sum_{i=1}^{N_A} U_{0i} \delta(x - x_i) \delta(y - y_i) [\eta(t - t_{1i}) - \eta(t - t_{2i})].$$

where N_A – quantity of the wells through which nitrogen is injected into the formation, U_{0i} – intensity of nitrogen injection through the i-th well $(i \in \{1, 2, \dots N_A\})$, $\delta(\dots)$ – Dirac's delta-function, $\eta(\dots)$ – Heaviside's function, x_i, y_i – coordinates of the i-th well, t_{1i}, t_{2i} – moments of commencement and ending of the process of gas injection through the i- th well.

Statement of problems. ***Identification problem*** [4]. Let's consider the formation Ω as the combination of n various interconnected formations Ω_i: $\Omega = \bigcup_{i=1}^{n} \Omega_i$. The multitude of the formations Ω is divided into the subset of the operating $\Omega^W = \{\Omega_1^W, \Omega_2^W, \dots, \Omega_{n^W}^W\}$ and buffer formations: $\Omega = \Omega^W \bigcup \Omega^B$, where n^W and n^B – quantity of the operating and buffer formations correspondingly.

We will consider that in any formation $\Omega_i (i = 1, \dots, n)$ there are the wells through which measurement of the formation pressure can be performed. The results of such measurement for the k-th well in the i-th formation can be given in the form of the time dependency $p_{ik}(t), t \in T_{ik}$, where T_{ik} – period of time during which the pressure was measured in the k-th well of the i-th formation.

These empiric data can be used together with the mathematical model (1)–(3) in order to determine the pressure propagation in the formations and identification of filtration of the formation properties. In particular, based on the formation pressure measurement data $p_{ik}(t)$ in the framework of the mathematical model (1)–(3) the following tasks can be considered:

to find change in time of the average value of the formation pressure $\bar{p}_i(t)$ for each formation $\Omega_i (i = 1, \dots, n)$ and determine dynamics of the gas flows between the formation and permeability of the interformation areas;

assuming that the powers h_i and porosity factors m_i of the formations Ω_i are known, to find factors of their permeability k_i;

subject to change of the middle formation pressures $\bar{p}_i(t)$, $(i = 1, \dots, n)$ to determine the volumes V_i of the gas accumulated in the formations Ω_i and using the gas state equation

$V_i = V_i(S_i, \overline{m}_i, \overline{h}_i, \overline{p}_i), S_i$ where $\overline{m}_i, \overline{h}_i, \overline{p}_i$ – surface area Ω_i, average porosity, power and pressure in the formations correspondingly, to ascertain the average parameters of the formations $\overline{m}_i, \overline{h}_i$ та $\overline{p}_i$.

It should be noted that there are no universal algorithms of identification of the collector-layer model parameters. It is connected with the complex geological structure of the formations, their heterogeneous propagation, usage of the two-dimension model, impossibility to divide effct of the parameters on the pressure propagation, irregularity of arrangement of the wells both operation and supervised impossibility to perform measurement of losses concurrently in all wells etc.

Buffer gas replacement problems. Let's assume that I_s – multitude of the wells existing in the blocks 1–3, and I_w – multitude of additional wells which, if necessary, can be installed at the gas storage facility. We will consider each of the multitudes as combination of two subsets $I_s = I_{sa} \cup I_{sn}$, $I_w = I_{wa} \cup I_{wn}$ where

I_{sa} – multitude of existing wells which can be used for nitrogen injection into the formation during the replacement process, I_{sn} – multitude of existing wells which can be used for nitrogen extraction from the formation during the replacement process, I_{wa} – multitude of additional wells which must be installed for nitrogen injection азоту into the formation, I_{wn} – multitude of additional wells which must be installed for nitrogen extraction from the formation during the process of replacement of the natural gas with nitrogen. We will designate the multitudes of the well coordinates consisting of the multitudes I_{wa} and I_{wn} as R_{wa} and R_{wn}: $R_{wa} = \{(x_i, y_i), i = 1, \dots, N_{wa}\}, R_{wn} = \{(x_j, y_j)$: where

$j = 1, \dots, N_{wn}\}$, N_{wa} – quantity of additional wells intended for gas injection, (x_i, y_i) – coordinates of these wells, – quantity of additional wells intended for gas extraction, (x_j, y_j) – coordinates of these wells. We will designate as $q_i^{sn}(t), i \in \{1, \dots, N_{sn}\}$ (N_{sn} – quantity of existing wells intended for gas extraction) intensity of extraction (loss) of the natural gas through the *i*-th well, and as $q_j^{wn}(t), j \in \{1, \dots, N_{wn}\}$ intensity of extraction (loss) through the *j*-th additional well which must be installed for the gas extraction. We will designate as $g_i^{sa}(t)$ $g_j^{sn}(t)$, $g_i^{wa}(t)$ та $g_j^{wn}(t)$ – losses of the fuel gas in the wells consisting of the multitudes I_{sa}, I_{sn},

I_{wa} and I_{sa} correspondingly. Let's assume that T – given period of time. Let's introduce the gas extraction functional Q_n, nitrogen injection functional Q_a and fuel gas loss functional Q_g during the process of replacement of the buffer gas in the formations with nitrogen

$$Q_n = \int_T \left(\sum_{i=1}^{N_{sn}} q_i^{sn}(t) + \sum_{j=1}^{N_{wn}} q_j^{wn}(t) \right) dt, \qquad (7)$$

$$Q_a = \int_T \left(\sum_{i=1}^{N_{sa}} q_i^{sa}(t) + \sum_{j=1}^{N_{wa}} q_j^{wa}(t) \right) dt, \qquad (8)$$

$$Q_g = \int_T \left(\sum_{i=1}^{N_{sn}} g_i^{sn}(t) + \sum_{j=1}^{N_{wn}} g_j^{wn}(t) + \sum_{i=1}^{N_{sa}} g_i^{wn}(t) + \sum_{j=1}^{N_{wa}} g_j^{wa}(t) + \right) dt \qquad (9)$$

It comes out from the mathematical model (1)–(6) that under the given initial conditions in all the formations Ω_i, given gas extraction modes $q_i^{sn}(t), i \in \{1, \dots, N_{sn}\}$ and $q_j^{wn}(t), j \in \{1, \dots, N_{wn}\}$, as well as given nitrogen injection modes $q_i^{sa}(t), i \in \{1, \dots, N_{sa}\}$ and $q_j^{wa}(t), j \in \{1, \dots, N_{wa}\}$ functionals Q_n, Q_a and Q_g depend only on division of the multitude I_s into the subsets I_{sa} and , as well as on the multitude of additional wells and

I_{sn} I_{wa} I_{wn}

And coordinates of their arrangement in the formations, which are determined by the multitudes R_{wa} and R_{wn}:

$$\begin{aligned} Q_n &= Q_n(I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{sa}, R_{sn}), \\ Q_a &= Q_a(I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{sa}, R_{sn}), \\ Q_g &= Q_g(I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{sa}, R_{sn}). \end{aligned} \qquad (10)$$

Let's formulate the problem of determination subject to the productive process of optimal extraction of the buffer gas with replacement thereof with nitrogen based on the given gas extraction modes $q_i^{sn}(t),\ \ i \in \{1, ..., N_{sn}\}$ and $q_j^{wn}(t),\ j \in \{1, ..., N_{wn}\}$ and nitrogen injection modes $q_i^{sa}(t),\ i \in \{1, ..., N_{sa}\}$ and $q_j^{wa}(t),\ j \in \{1, ..., N_{wa}\}$ determine the multitudes $I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{sa}, R_{sn}$, according to which the gas extraction functional Q_n, calculated using the mathematical model (1)–(6) achieves the maximal value

$$\mathbf{Q} = \max_{I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{wa}, R_{wn}} (\mathbf{Q}_n) \qquad (11)$$

subject that the fuel gas loss functional does not exceed the given value Q_g^*:

$$Q_g\,(I_{sa}, I_{sn}, I_{wa}, I_{wn}, R_{wa}, R_{wn}) \le Q_g^*, \qquad (12)$$

and the nitrogen maximal concentrations $C_i^{\max},\ i \in \{1, 2, ..., n^W\}$ in the working seam does not exceed the given value Ci*

$$C_i^{\max},\ i \in \{1, 2, ..., n^W\}$$

let's assume that the multitude of the well I_s and its subsets I_{sa} and I_{sn} are given: $I_s = I_{sa} \cup I_{sn}$. In this case we can formulate the problems of optimal regulation of the buffer gas extraction and replacement modes.

Let's designate as $\mathbf{q}_a(t) = \left[q_1^{sa}(t), q_2^{sa}(t), ..., q_{N_{sa}}^{sa}(t)\right]^{\mathrm{T}}$ the set of the functions determined in the period of time $\boldsymbol{T_a}$, which determine the modes of nitrogen injection into the formation through the wells I_{sa}.

$\mathbf{q}_n(t) = \left[q_1^{sn}(t), q_2^{sn}(t), ..., q_{N_{sn}}^{sn}(t)\right]^{\mathrm{T}}$ - set of the functions determined in the period of time T_n, which determine the modes of extraction from the formation through the wells I_{sn}. As it comes out from the mathematical model (1)–(6) based on the given multitude $I_s = I_{sa} \cup I_{sn}$, the functionals

Q_n, Q_a and Q_g depend only on the initial conditions and modes of operation of the wells I_{sa} and I_{sn}:

$$Q_n = Q_n(q_a(t), q_n(t)),\ Q_a = Q_a(q_a(t), q_n(t)),\ Q_g = Q_g(q_a(t), q_n(t)). \ (14)$$

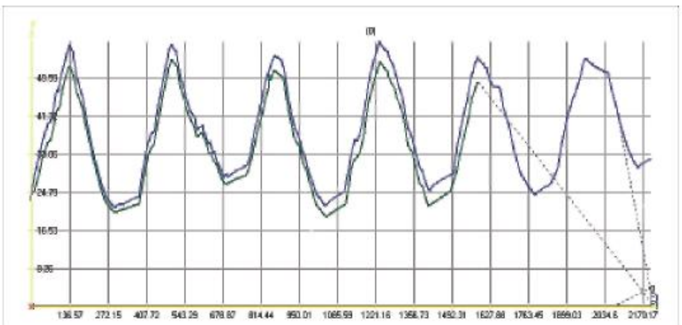

Fig. 2. Formation pressure in the well 165 of the block 4 and in the operation area within six seasons of the gas extraction and injection

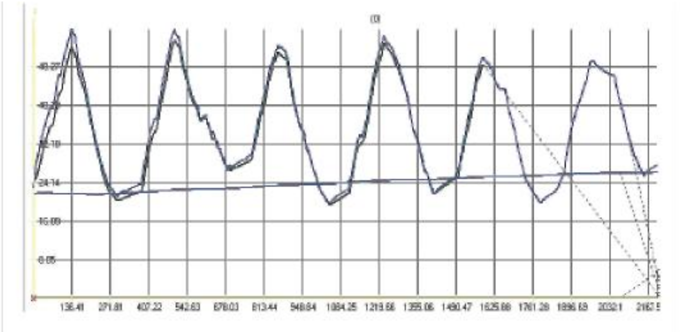

Fig. 3. Estimated formation pressure in the operation area (blue curve) during nitrogen injection into the top of the block 3 with the intensity of 2.9 m[3]/sec. within 4 years (1,460 days)

Let’s formulate now the problem of determination of the mode of optimal nitrogen injection based on the given multitude of the wells $I_s = I_{sa} \cup I_{sn}$: for the given set of functions $q_n(t)$ which determine the modes of operation of the wells I_{sn} to determine the set $q_a(t) \in M_{q(t)}$ of the functions which determine the modes of operation of the wells I_{sa} subject to which the functional Q_a achieves the maximal value Q

$$Q = \max_{q_a(t)}(Q_n), \qquad (15)$$

where $M_{q(t)}$– multitude of all possible sets of functions $q_a(t)$ which determine the modes of operation of the wells I_{sa} for which the maximal nitrogen concentrations $C_i^{\max}, i \in \{1, 2, ..., n^W\}$ in the operating formations calculated using the mathematical model (1)–(6) do not exceed the given values C_i^*.

Let’s assume that $q_w(t) = [q_1^{wn}(t), q_2^{wn}(t), ..., q_{N_{wn}}^{wn}(t)]^T$ – set of functions given in the period of time T^W which determine the modes of the gas extraction by the wells I_W in the operating formations, $q_a(t) = [q_1^{ba}(t), q_2^{ba}(t), ..., q_{N_{ba}}^{ba}(t)]^T$ – set of functions given in the period of time T^{Ba} which determine the modes of nitrogen injection through the wells I_{Ba} in the buffer formations, $q_n(t) = [q_1^{bn}(t), q_2^{bn}(t), ..., q_{N_{bn}}^{bn}(t)]^T$ - set of functions given in the period of time T^{Bn} which determine the modes of the buffer gas extraction from the wells I_{Bn} in the buffer formations.

Let’s formulate now the problem of optimal control of the modes of replacement of the buffer gas with nitrogen in the operated formation: based on the given set of functions $q_w(t)$ which determine the modes of operation of the wells in the operating formations Ω^W, to find the sets of functions $q_a(t)$ and $q_n(t)$ subject to which the functional Q_n calculated for the wells I_{sn} in the buffer formations achieves its maximal value Q

$$Q = \max_{q_a(t), q_n(t)}(Q_n) \qquad (16)$$

the least of all possible interval T^{Ba} гof nitrogen injections and subject that the calculated using the model (1)–(6) maximal nitrogen concentrations in the operating formations Ω^W do not exceed the given values.

Algorithm of calculation of the nitrogen propagation line coordinates. Calculation of the line of nitrogen propagation without mixing its with the natural gas.

Let’s consider a heterogeneous formation according to permeability, porosity and power. Let’s assume that the nitrogen propagation process happens without its mixing with the natural gas, i.e. separate filtration of two gases is considered. Nitrogen is injected through some wells, and through other ones extraction of the natural gas is possible. According to power (the difference of elevation points of the top and bottom surfaces of the formation) the collector-layer as compared to other sizes is insignificant. The geometric sizes of collector-layers achieve hundreds and thousands of meters, and the filtration processes are studied in significant time intervals (moths and years). In these assumptions the relation of the capillary pressure to the full hydrodynamic loss of pressure is small. It allows neglecting the capillary forces. The gas flow is subordinated by the Darcy's law. The gravitation forces are not taken into account.

The gas extraction (injection) in underground gas storage facilities is performed through n wells located in the points (x_i, y_i) within a certain period of time $t \in [t_{1i}, t_{2i}], (i = \overline{1,n})$. The density of extraction is determined using the formula

$$q(t) = \frac{1}{V}\sum_{i=1}^{n} q_i \delta(x - x_i)(y - y_i)[\eta(t - t_{1i}) - (t - t_{2i})], \qquad (17)$$

where q_i – gas extraction from i-th well, $\delta(x)$ – Dirac's delta-function, $\eta(t - t_{ji})$ – Heaviside's unit function, V – gas storage facility volume.

The multitude of all wells S is combination of two subsets of the wells S_1 and S_2. The multitude S_1 consists of the operating wells, and S_2 – wells through which nitrogen is injected. In this view, the collector-layer area is also divided into two multitudes of areas. In one of the area multitudes nitrogen is present. In the nitrogen propagation area the following nitrogen state equation is true

$$P = g\rho_a z_a R_a T, \qquad (18)$$

And in the external – the following natural gas state equation is true

$$P = g\rho z R T. \qquad (19)$$

where F – nitrogen arrangement area.

On the edge "nitrogen-natural gas" the pressure equality condition is true. The condition as to the gas volume in the internal area must also be taken into account.

$$Q_\Sigma = \frac{T_{cr}}{P_{cr}}\int_0^F\int_0^h \frac{pm}{Tz} dF dh \approx \frac{T_{cr}}{P_{cr}} \frac{\bar{p}}{\overline{Tz}} \overline{mh} F, \qquad (20)$$

In order to find the line point coordinates the following actions must be taken: at every moment of time the pace of movement of the points $(x, y) \in \Gamma_a$ of the line must be found

$$\bar{v}(x, y, t)_{\bar{n}} = -\frac{k}{\mu_a}\frac{dp}{d\bar{n}}$$

based on the pressure gradient along the normal $\bar{n}$ up to the line delimiting the natural gas and the nitrogen and natural gas mixture, where $\bar{v}$ – filtration pace vector in the direction of the normal in the point (x, y) on the line Γ_a, k – permeability factor, μ_a – nitrogen dynamic viscosity, p – reduced pressure. In case of change of the line length, the density of points $(x, y) \in \Gamma_a$ on the line must be kept stable. In order to accelerate the time of calculation of the line parameters, the density of points on the line may be increased gradually.

One must continuously control the equation $V_a(t) = \hat{V}_n(t)$. where $V_a(t)$ – volume of the gas located in $\Omega_a(t, \Gamma_a)$, and – total gas volume which

$$V_n = \sum_{i=1}^{n} V_i(t)$$

came into the collector-layer for the time t through n injection wells. In case if the estimated nitrogen volume according to the calculated line is not equal to the injected nitrogen volume (which is calculated based on the parameters of lumped wells), then adjustment of the pace of movement of points on the line is performed in such a way so that to achieve the equality (10) with the given accuracy.

If adjustment of the propagation rate on each time pace will be significant then it can be ascertained:

$$\bar{v}(x,y,t)_{\bar{n}_a} = \begin{cases} -\dfrac{k_a}{\mu_a}\dfrac{\Delta p_a}{\Delta \bar{n}_a} - \dfrac{k}{\mu}\dfrac{\Delta p}{\Delta \bar{n}}, & \dfrac{\Delta p_a}{\Delta p} \geq 0; \\ -0{,}5\left(\dfrac{k_a}{\mu_a}\dfrac{\Delta p_a}{\Delta \bar{n}_a} - \dfrac{k}{\mu}\dfrac{\Delta p}{\Delta \bar{n}}\right), & \dfrac{\Delta p_a}{\Delta p} \leq 0., \end{cases} \quad (21)$$

where the first summand is located in the internal area (nitrogen location area), and the seconf summand – in the external area. For the time Δt the point $(x, y) \in \Gamma_a$ in the direction of the normal will pass the path $\Delta t v(x, y, t + \Delta t)$.

During nitrogen injection into several wells, the quantity of unconnected areas filled with nitrogen continuously changes.

Numeric Experiments

Experiment 1. Performance of numeric experiments was preceded by adaptation of the mathematical model of calculation of the nitrogen propagation area to real gasodynamic and filtration processes which occur in heterogenic porous areas (collector-layers). The adaptation process consisted in finding of the parameters of permeability of formation of the gas storage facility, its separate blocks and low-permeable interlayers between separate formations. Experimenting on the program complex the adapted parameters were continuously ascertained. One of the important and sufficiently satisfactory arguments as to the model is high accuracy of the calculated parameters of the pressure change dynamics in the neutral period (between completion of the gas extraction and commencement of the gas injection).

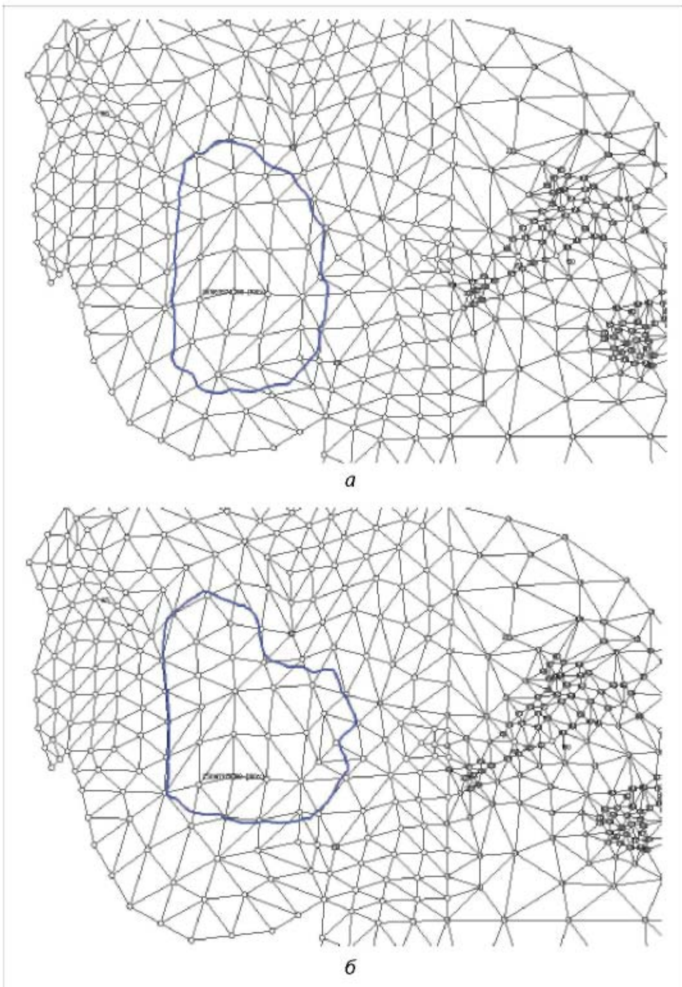

Fig. 4. Nitrogen propagation line on the 600th day after injection of 260 mln m^3 of nitrogen during 600 days (5 m^3/sec.) (*a*); nitrogen propagation line on the 900th day after injection of 260 mln m^3 of nitrogen during the first 600 days (5 m^3/sec.) (*б*)

Calculation of the formation pressure in the operating area was performed within six seasons of the gas injection and extraction. You can evaluate the adaptation results based on the diagrams of the measured and calculated formation pressures in the operating area which are given in Fig. 3. Insignificant deviation of the calculated and measured formation pressures at the initial moment (the first season of injection) occurs based on insufficient balance of the model and real parameters, as well as as a result of different approaches to calculation of the formation pressures (the program allows calculating of the average pressure in the operating area for all operating wells).

It should be noted that the pressure change dynamics in the well 165 of the block 4 is close to real.

Experiment 2. Nitrogen injection was performed in the block 3. It is connected with the fact that the block 4 has sufficiently low permeability of the formation. Several variants were studied. Nitrogen injection was performed with different intensities. Fig. 3 shows the variant of nitrogen injection into the block 3 with the intensity of 2.9 m^3/sec. Nitrogen injection effect was evaluated based on change of the formation pressure in the neutral period after the gas extraction.

Visually analyzing the diagram in Fig. 3, you can notice tangible changes of the formation pressure in the operating area (see pressures in the neutral periods).

Table 1 Pressure change (MPa) in the neutral period after injection of 182.9 mln m^3 of nitrogen

End of the *i*–th season of the natural gas extraction	After nitrogen injection	Without nitrogen injection	Pressure difference
i = 2	2.868	2.777	**0.091**
i = 3	2.265	2.116	**0.149**
i = 4	2.558	2.347	**0.211**

Table 2 Pressure change (MPa) in the neutral period after injection of 365.8 mln m^3 of nitrogen

End of the *i*–th season of the natural gas extraction	After nitrogen injection	Without nitrogen injection	Pressure difference
i = 2	2.975	2.777	**0.198**
i = 3	2.397	2.116	**0.281**
i = 4	2.744	2.347	**0.397**

***Experiment* 3.** Nitrogen injection into separate wells of the second area was performed during operation of the gas storage facility.

***Experiment* 4.** Nitrogen injection into 4th area was performed within 5 years by four wells with the intensity of 0.7408 m/sec^3 (the total nitrogen volume made up 467.237376 mln m^3).

The pressure difference value in the neutral periods calculated within 5 years of operation of the Dashavske UGSF is given in Table 3.

Таблиця 3 Pressure change in the operating area during the nitrogen injection process (without extraction of the natural gas displaced from the low-permeable areas)

Year	MPa	Increase of the formation pressure in the operating area within the season
1	0.0662	0.0662
2	0.1619	0.0957
3	0.2651	0.1032
4	0.3829	0.1178
5	0.5375	0.1546

Concurrently with nitrogen injection into the 4th area which was performed within 5 years by four wells with the intensity of 0.7408 m/sec^3 (the total nitrogen volume made up 467.237376 mln m^3), the gas was additionally extracted from the UGSF operating area by three wells with the intensity of 0.75 m^3/sec (the total nitrogen volume made up 354.78 mln m^3).

If the pressures in the operating area at the end of the fifth season based on the found gas extraction in addition to the existing one coincided, then it confirms regularity of the found natural gas volumes displaced by nitrogen into the operating area.

Numeric experiment results

1. The gas volumes which will be displaced into the operating area for each season are proportionate to change of the pressures in the operating area for the season (the pressure difference subject to absence of nitrogen injection and under the injection conditions), i.e. as of the end of: the first season – 0.0662, the second season – 0.0957, the third season – 0.1032, the fourth season – 0.1178, the fifth season – 0.1546 MPa.

As we see from the given table, the dynamics of flow of the gas displaced from low-permeability areas into the operating area increases over the years.

2. For five seasons of the natural gas extraction and injection as a result of injection of nitrogen with the volume of 467.237376 mln m^3 (with the intensity of 256 thsd. m^3 per day within five years), 354.78 mln m^3 of the natural gas were displaced by nitrogen into the operating area from the low-permeable areas.

3. The buffer gas replacement process is possible. It will be connected more likely with the pressure growth and existing formation anisotropy rather than with the nitrogen line movement. It can be expected that during the line movement nitrogen propagation more than 30 % of the buffer natural gas will get into the internal nitrogen location area.

4. During the neutral period about 130 mln m^3 of gas get into the operating area (operating well area), and about 200–250 mln m^3 of gas – to the whole operating area depending on the gas injection and extraction volumes. Much more gas gets into the operating area during the gas extraction period. Stoppage of the gas flow from the operating into the third and forth areas can provide 92.08 mln m^3 of nitrogen. For the gas extraction season amounting at average to 151 days, replenishment of the buffer gas allows saving 695.5 thsd m^3 of the fuel gas.

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